Express the following riemann sum as a definite integral

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\frac{9(i-1)}{n}}\ \frac{9}{n} \qquad \Delta x = \frac{9}{n} \qquad \sum_{i=1}^{n} f(x_a + i\Delta x)\Delta x = R(n)$$

$$x_a + i\Delta x = \frac{9(i-1)}{n} = \frac{9i}{n} - \frac{9}{n} = -\frac{9}{n} + \frac{9i}{n} = \underset{x_a}{-\frac{9}{n}} + \underbrace{i\frac{9}{n}}_{\Delta x}$$

$\Rightarrow x_a = -\frac{9}{n}$ is not in constant form, for it depends on n! Hint: change $i-1$ to i, Thus...

- Increase i by 1, decrease in start value

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\frac{9(i-1)}{n}}\ \frac{9}{n} = \lim_{n\to\infty} \sum_{i=0}^{n-1} \sqrt{\frac{9i}{n}}\ \frac{9}{n}$$

- Start with $i=1$ and compensate

$$= \lim_{n\to\infty} \sum_{i=1}^{n-1} \sqrt{\frac{9i}{n}}\ \frac{9}{n} + \sqrt{\frac{9(0)}{n}}\ \frac{9}{n}$$

- Increase upper limit by 1 and compensate

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\frac{9i}{n}}\ \frac{9}{n} - \sqrt{\frac{9(n)}{n}}\ \frac{9}{n} + \sqrt{\frac{9(0)}{n}}\ \frac{9}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\frac{9i}{n}}\ \frac{9}{n} - \underbrace{\sqrt{9}\ \frac{9}{n}}_{\to\ 0 \text{ as } n\to\infty} + 0$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\underbrace{\frac{9i}{n}}_{\underset{x_a}{0} + \underset{\Delta x}{\frac{9}{n}} i}}\ \frac{9}{n}$$; this is a nice form allowing x_a

$x_b - x_a = 9$

$x_b - 0 = 9$

$x_b = 9$

$$= \int_0^9 \sqrt{x}\, dx$$