Integration By Parts - An Interesting Example

(from p. 201 of 'Calculus DeMystified')
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Evaluate $\int_{\pi/2}^{2\pi} \sin x \cos x \, dx$

Recall $\int u\,dv = uv - \int v\,du$

Let $u = \sin x$; $dv = \cos x\,dx$

$\int \sin x \cos x\,dx = \sin x \sin x - \int \sin x \cos x\,dx$

The initial expression appears in the result of integration by parts, but thats ok!

Rearranging ...

$\int \sin x \cos x\,dx + \int \sin x \cos x\,dx = \sin^2 x$

$2\int \sin x \cos x\,dx = \sin^2 x$

$\int \sin x \cos x\,dx = \frac{\sin^2 x}{2} \Rightarrow \int_{\pi}^{2\pi} \sin x \cos x\,dx = \frac{1}{2} \sin^2 x \Big|_{\pi}^{2\pi}$

$= \frac{1}{2}\left[\sin^2 2\pi - \sin^2 \frac{\pi}{2}\right]$

$= \frac{1}{2}\left[0 - 1\right] = -1/2$