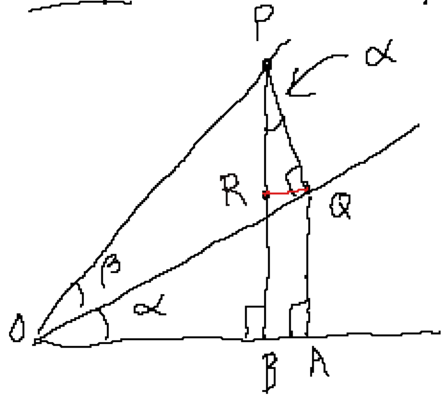
Proof of: $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

Step 1 Draw the picture & label



- draw angle α
- draw angle β
- mark P. Let $OP = 1$
- drop $\perp$ from P to B
- drop $\perp$ from P to Q
- the $\angle$ BPQ is: α
- drop $\perp$ from Q to A.
- draw $RQ \parallel AB$

$OP = 1$ // Given

$PQ = ? \Rightarrow \sin\beta = \frac{PQ}{OP} = \frac{PQ}{1} \Rightarrow PQ = \sin\beta$

$OQ = ? \Rightarrow \cos\beta = \frac{OQ}{OP} = \frac{OQ}{1} \Rightarrow OQ = \cos\beta$

$RQ = ? \Rightarrow \sin\alpha = \frac{RQ}{PQ} \Rightarrow RQ = \sin\alpha PQ = \sin\alpha\sin\beta$

$\cos(\alpha+\beta) = \frac{OB}{OP} = \frac{OB}{1} = OB = OA - BA$

But $BA = RQ$

$= OA - RQ$

$= \cos\alpha\cos\beta - RQ$

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$

Thank you to:

https://en.wikipedia.org/wiki/Proofs_of_trigonometric_identities

RayByczko, 2023-3-24